

The realistic models of relativistic stars in $f(R) = R + \alpha R^2$ gravity

Artyom V. Astashenok*,¹ Álvaro de la Cruz-Dombriz†,² and Sergei D. Odintsov‡^{3,4}

¹*I. Kant Baltic Federal University, Institute of Physics,*

Mathematics and IT, Nevskogo st. 14, 236016 Kaliningrad, Russia

²*Cosmology and Gravity Group, Department of Mathematics and Applied Mathematics,
University of Cape Town, Rondebosch 7701, Cape Town, South Africa.*

³*Institució Catalana de Recerca i Estudis Avançats (ICREA), Barcelona, Spain*

⁴*Institute of Space Sciences (IEEC-CSIC) C. Can Magrans s/n, 08193 Barcelona, Spain*

In the context of $f(R) = R + \alpha R^2$ gravity, we study the existence of neutron and quark stars for various α with no intermediate approximation in the system of equations. Analysis shows that for positive α the scalar curvature does not drop to zero at the star surface (as in General Relativity) but exponentially decreases with distance. Also the stellar mass bounded by star surface decreases when the value α increases. Nonetheless distant observers would observe a gravitational mass due to appearance of a so-called *gravitational sphere* around the star. The non-zero curvature contribution to the gravitational mass eventually is shown to compensate the stellar mass decrease for growing α 's. We perform our analysis for several equations of state including purely hadronic configurations as well as hyperons and quark stars. In all cases, we assess that the relation between the parameter α and the gravitational mass weakly depend upon the chosen equation of state. Another interesting feature is the increase of the star radius in comparison to General Relativity for stars with masses close to maximal, whereas for intermediate masses $1.4 - 1.6 M_\odot$ the radius of star depends upon α very weakly. Also the decrease in the mass bounded by star surface may cause the surface redshift to decrease in R^2 -gravity when compared to Einsteinian predictions. This effect is shown to hardly depend upon the observed gravitational mass. Finally, for negative values of α our analysis shows that outside the star the scalar curvature has damped oscillations but the contribution of the *gravitational sphere* into the gravitational mass increases indefinitely with radial distance putting into question the very existence of such relativistic stars.

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I. INTRODUCTION

The discovery of the accelerated expansion of the universe [1, 2] has recently led to an intensive study of cosmological applications of extended (also dubbed modified) theories of gravity which aim to overcome the limitations of the cosmological Concordance Λ CDM model, made of General Relativity (GR), a cosmological constant Λ and Cold (non-baryonic) Dark Matter. Many of these extensions aim to avoid the commonly invoked need for dark components which is required, for instance both in the Λ CDM model, where dark energy is nothing but a cosmological constant, and in phantom/quintessence-like models where a scalar field is needed, among others. Throughout the years, a plethora of modified gravity theories have been shown to be able to preserve the positive results of Einstein theory and obtain the required cosmological evolution with the most relevant cosmological epochs [3–5]. Among those attempts, one of the most successful ones is the so-called $f(R)$ theories where the gravitational Lagrangian includes powers of the Ricci scalar R encompassed in an arbitrary function of R (*c.f.* [6–8] for extensive reviews on the subject).

Since in principle the number of viable modified gravities, and classes of models therein, is sufficiently large, the comparison between theoretical predictions and different catalogues of cosmological dataset, extracted from both the cosmic background and relativistic perturbations (mainly scalar and tensor) evolutions, remains mandatory. Usually, such comparisons suffer from the so-called degeneracy problem, meaning that several competitive gravitational theories are able to explain the same phenomena with roughly speaking the same statistical precision. Other approaches recently explored have considered model-independent (cosmographic) techniques in order to reconstruct the underlying theory capable of causing the observed cosmological evolution once the Copernican principle is assumed (*c.f.* [9, 10] and references therein).

Thus, in order to further constrain the number of experimentally viable theories beyond Einsteinian gravity and obtain more rigid constraints on the parameters space of every theory, one possibility involves studying the consequences of these proposals at an astrophysical level. Understandably, these high-energy configurations are the ideal arena to study the behaviour of extended theories of gravity in strong gravitational field regimes and may complement the picture as provided by the low-curvature late-time cosmological evolution. First of all, it is interesting to investigate the possibility of the existence and features of black holes [11] and relativistic stars, such as white dwarfs and neutron stars, the latter

*aastashenok[at]kantiana.ru

†alvaro.delacruzdombriz [at] uct.ac.za

‡odintsov[at]ieec.uab.es

being the objective of the research in this paper. Thus for viable classes of $f(R)$ Lagrangians, one needs to obtain the relations between the paradigmatic parameters of relativistic stars such as masses, radii, momenta of inertia, quadrupole momenta and Love number, among others. Unfortunately, at present, fully precise astrophysical measurements cannot be provided for the aforementioned quantities, the exception being the mass measurements for neutron stars.

As extensively studied, once GR is assumed as the correct theory of gravity, there are certainly upper limits for realistic neutron star masses, being the theoretical mass limit, itself having increased over the years, from $\mathcal{O}(0.6M_\odot)$ that Tolman-Oppenheimer-Volkoff (TOV) [12] found for a free neutron gas equation of state (EoS), to $\mathcal{O}(2.2M_\odot)$ limits when stiffer chiral interactions are allowed [13]. Notwithstanding, the discovery of a neutron star with mass $1.974M_\odot$ [14] by the Shapiro delay method, confirmed some years after by a precise sighting [15] through binary system measurements, have indeed put into question the validity of GR in such a high-energy gravitational environment and have been able to rule out exotic EoS proposals. Further observations have certainly confirmed the existence of neutron stars heavier than the GR naive prediction [16]. In that realm, some attempts have also tried to reconcile these results with GR, assuming for instance strong magnetic fields [17] or electric fields [18] inside the star.

Neutron stars in the context of $f(R)$ theories were primarily studied in [19, 20] and then in abundant bibliography [21–34] for both scalar-tensor $f(R)$ theories, as well as some closely-related theories. Also the possibility of the emergence of new types of astrophysical objects in the context of $f(R)$ theories, such as stable stars with large magnetic fields, supermassive stars, among others (see for instance [35, 36] for some proposals).

Perturbative solutions of TOV equations for simple $f(R)$ models were investigated in [17, 37, 38]. Obviously, limitations of a perturbative approach lie mainly in the impossibility of comparing the (unknown) exact solution with the determined perturbative solution. Also one would expect that in strong gravity regimes, non-GR gravitational effects to be dominant and consequently, the GR limit cannot be simply considered as the leading contribution to the solution. Thus, for a careful description one needs to solve exact higher-order differential equations as we shall do herein.

Another approach in the literature has involved resorting to the equivalence between $f(R)$ theories and Brans-Dicke scalar-tensor theory in order to study realistic models of neutron and strange stars [39, 40]. Within this approach the description in terms of $f(R) = R + \alpha R^2$ gravity was proposed in [41] and illustrated for quark stars. There, it was shown that for distant observers the gravitational mass of star increases with increasing α ($\alpha > 0$), though the interpretation of this fact depends upon the frame in which calculations are performed.

Considering $f(R)$ theory directly, one can see that the

matching conditions at the edge of the star, do not impose the scalar curvature to vanish [42], so $R \neq 0$ is in principle expected outside the star. As a consequence, the spacetime region outside the star also contributes to the total gravitational mass as perceived by a distant observer (*c.f.* [31] for a thorough discussion on this issue). By analogy with scalar-tensor theory of gravity in which that region outside the star is referred to as the dilaton sphere (or disphere) [43], in the following we shall denote this domain as *gravitational sphere* (or *gravisphere*).

In this paper we shall consider in detail the possibility of existence and the main features of neutron stars and quark (also dubbed strange) stars in the context of $f(R) = R + \alpha R^2$ gravity. The TOV set of equations for $f(R)$ theories is presented in the Section II. Then we shall briefly discuss the various equations of state (EoS) for nuclear matter in Sec. III. As mentioned above, for modified gravity theories, in principle one needs to investigate the solution of field equations outside the star since it may occur that the exterior spacetime is not Ricci flat and Schwarzschild is not the only permitted solution. As we shall show, the obtained exterior solution crucially depends upon the chosen EoS because the latter fixes the scalar curvature value at the star surface. The next two following section will be devoted to obtaining solutions for various EoS. In Sec. IV we shall show how for positive α one needs to choose the initial condition for $R(0)$ to satisfy the asymptotic (Schwarzschild) flatness requirement. The scalar curvature outside the star thus sharply decreases. It will be shown that the stellar mass, for a given central density (i.e., density at the centre of the star) is smaller than the GR counterpart. Nonetheless the emergence of the *gravitational sphere* around the star, provides an extra contribution to the gravitational mass as measured by a distant observer. As a result the gravitational mass overcomes the GR counterpart for central density values higher than a given critical central density. The value of this critical central density will be shown to be determined by both α and the chosen EoS. This effect takes place for various EoS, including the exotic case of quark stars. It is interesting to note that the values of radii for intermediate masses $1.4M_\odot < M < 1.6M_\odot$ very weakly depend upon α . Finally, in Sec. V we shall show how for $\alpha < 0$ cases, although outside the star the scalar curvature possess damped oscillations, the gravitational mass contribution when taking into account the increases with the radial distance. We conclude the paper by showing our main conclusions in Sec. VI. Figures for all the studied cases are presented at the end of the paper and are intended to help the reader to understand the explained trends in the bulk of the paper.

II. EQUATIONS FOR COMPACT STARS IN $f(R)$ GRAVITY

Let us consider the gravitational action for $f(R)$ gravity where R denotes the scalar curvature

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R), \quad (1)$$

where $G = c = 1$ units will be used throughout the paper. The corresponding field equations can be obtained by varying action (1) with respect to the metric $g_{\mu\nu}$, yielding

$$f_R G_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^\alpha \nabla_\alpha) f_R - \frac{1}{2} (f(R) - R f_R) g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad (2)$$

where $f_R \equiv df(R)/dR$, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is Einstein tensor and $T_{\mu\nu}$ is energy-momentum tensor, which for a perfect fluid in a comoving frame can be written as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}. \quad (3)$$

As easily deduced taking the trace of Eq. (2), in GR ($f(R) \equiv R$) the scalar curvature is univocally defined by the energy-momentum content through an algebraic equation. On the contrary, in the context of $f(R)$ theories of gravity in the metric formalism, given the higher order of the field equations, R may be thought of as an independent dynamical function.

For a spherically symmetric metric of the form

$$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (4)$$

the corresponding field equations as obtained from Eq. (2) for functions A and $\psi = B'/B$ (prime denotes derivative with respect to r) become

$$A' = \frac{2rA}{3f_R} \left[8\pi A(\rho + 3p) + Af(R) - f_R \left(\frac{A}{2} R + \frac{3\psi}{2r} \right) - \left(\frac{3}{r} + \frac{3\psi}{2} \right) f_{2R} R' \right], \quad (5)$$

$$\psi' = \frac{\psi}{2} \left(\frac{A'}{A} - \psi \right) + \frac{2A'}{rA} + \frac{2}{f_R} \left[-8\pi A p + \left(\frac{\psi}{2} + \frac{2}{r} \right) \times f_{2R} R' - \frac{A}{2} f(R) \right], \quad (6)$$

$$p' = -\frac{\rho + p}{2} \psi, \quad (7)$$

and the trace of Eq. (2) gives the equation for the scalar curvature:

$$R'' = R' \left(\frac{A'}{2A} - \frac{\psi}{2} - \frac{2}{r} \right) - \frac{A}{3f_{2R}} [8\pi(\rho - 3p) + f_R R - 2f(R)] - \frac{f_{3R}}{f_{2R}} R'^2, \quad (8)$$

Finally one needs to specify the $f(R)$ model under consideration (in our case $f(R) = R + \alpha R^2$, with α is

constant parameter) and add the EoS for completeness of the system, i.e., $p = f(\rho)$. Several EoS will be considered and explained in Sec. III.

Initial and boundary conditions: The following conditions at the centre of star ($r = 0$) should be imposed in order to guarantee regularity and finiteness both of density and pressure everywhere. This provides

$$A(0) = 1, \psi(0) = 0, R'(0) = 0. \quad (9)$$

Therefore there is only one free initial condition for the system of equations Eqs. (5)-(8), namely the value of the scalar curvature $R(0)$. At the surface of star, both pressure and density (for most EoS) cancel, i.e., $\rho_s = p_s = 0$. Thus, from the star radius onwards, one needs to integrate our system of Eqs. (5)-(8) with $\rho = p = 0$. The boundary conditions at spatial infinity can be determined from the reasonable requirement that the star exterior solution should be matched to the Schwarzschild one at infinity. Therefore

$$R \rightarrow 0, \quad A \rightarrow 1, \quad r \rightarrow \infty. \quad (10)$$

From the equations above, one can see that solutions for $A(r)$, $\rho(r)$, $p(r)$ and $\psi(r)$ do not depend on the chosen value $B(0)$. However, in order to achieve, as desirable, Schwarzschild solution at the asymptotic spatial infinity, one must require $B(r) \rightarrow 1$ for $r \rightarrow \infty$. If $B(r) \rightarrow B_\infty \neq 1$ at $r \rightarrow \infty$, one can simply rescale the initial condition for $B(0)$ as

$$B(0) \rightarrow \frac{B(0)}{B_\infty}. \quad (11)$$

On the other hand, one usually assumes $A(r)$ to be of the form

$$A(r) = \left(1 - \frac{2m(r)}{r} \right)^{-1} \quad (12)$$

and therefore the mass function $m(r)$ as a function of radial coordinate can be expressed as :

$$m(r) = \frac{r}{2} (1 - A(r)^{-1}). \quad (13)$$

Mass extraction for negative α : For the performed analysis with $\alpha < 0$, we have seen that $A(r)$ oscillates around unity at sufficiently large distances and therefore one cannot extract the gravitational mass from this parametrization correctly. For $r \rightarrow \infty$ we have $B \sim e^{2\phi}$ where ϕ can be interpreted as the Newtonian potential for the gravitational field at distance r , in other words, $\phi(r) = -m(r)/r$. Therefore $B(r) \approx 1 - \frac{2m(r)}{r}$ at sufficiently large distance r . Provided the gravitational mass has a well-defined limit one can assume that $m(r) \approx M = \text{const}$ from some $r = r_c$ onwards (r_c being a sufficiently large distance) and thus, the gravitational mass M can be extracted from $B(r)$. Consequently for

an initial value of $B(0)$ so that $B(r) \rightarrow 1$ at $r \rightarrow \infty$ we have

$$B(r) = 1 - \frac{2m(r)}{r}, \quad (14)$$

and therefore the gravitational mass M can be calculated as the limit

$$M \equiv \lim_{r \rightarrow \infty} r(1 - B(r))/2. \quad (15)$$

Mass extraction for positive α : As we shall see in Sec. IV, in the event $\alpha > 0$, the limit $m(r) \rightarrow M$ at spatial infinity is well defined. In the event that the value of $B(0)$ is arbitrarily chosen, one first needs to obtain the asymptotic value $B_\infty = \lim_{r \rightarrow \infty} B(r)$, so that function $B(r)$ becomes

$$B(r) = B_\infty \left(1 - \frac{2m(r)}{r} \right), \quad (16)$$

so a simple relation for gravitational mass

$$M = \frac{1}{B_\infty} \lim_{r \rightarrow \infty} \frac{r(B_\infty - B(r))}{2}, \quad (17)$$

is obtained.

We need carefully investigate the possibility of appearance of contributions with r^γ ($\gamma > 0$) in $B(r)$. In this case we have no finite limit for mass. The behaviour of the resulting solution for the metric functions A and B in the outer region depends values $A(r)$, $B(r)$ and its derivatives on star surface ($r = r_s$). For the latter, specific values depend upon the chosen EoS.

III. REALISTIC EQUATIONS OF STATE

For our investigation we have considered a representative set of four realistic EoS.

Neutron stars: Below we provide the main features and assumptions in the EoS under study herein.

- 1) the well-known SLy4 EoS [44, 45] is obtained from many body calculations with simple two-nucleon potential. For completeness, we have also considered the AP4 EoS [46] which hosts a three-nucleon potential and Argonne 18 potential with UIX potential;
- 2) as an example of EoS based on relativistic mean-field (RMF) calculations we take the GM1 model firstly considered by Glendenning and Moszkowski (GM) [47]. We also consider a the generalization of GM1 model, the so-called GM1nph [48] for cold neutron star matter in β -equilibrium containing the baryon octet and electrons;
- 3) relatively stiff MPA1 EoS [49] obtained from relativistic Dirac-Brueckner-Hartree-Fock formalism. This EoS accounts for the energetic contributions originated by the exchange between π and ρ mesons.

On Fig. 1 we plot dependence of pressure from energy density for these four EoS. One can see that for intermediate densities these EoS seem indistinguishable. For GM1 EoS there is a softening effect at high densities due to appearance of hyperons (mainly Σ^- , Λ^0 and Σ^0). In our calculations we have used analytical representations of these EoS. Thus, the SLy EoS can be represented as

$$\zeta = \frac{a_1 + a_2\xi + a_3\xi^3}{1 + a_4\xi} f(a_5(\xi - a_6)) + (a_7 + a_8\xi) \times f(a_9(a_{10} - \xi)) + (a_{11} + a_{12}\xi)f(a_{13}(a_{14} - \xi)) + (a_{15} + a_{16}\xi)f(a_{17}(a_{18} - \xi)), \quad (18)$$

where $\zeta \equiv \log(P/\text{dyn cm}^{-2})$, $\xi \equiv \log(\rho/\text{g cm}^{-3})$, and $f(x) \equiv 1/(e^x + 1)$. The coefficients a_i can be found in [50]. For MPA1, AP4 and GM1 (with hyperons) EoS take the more complicated dependence shown in [51], namely

$$\zeta = \zeta_{low} f(a_1(\xi - c_{11})) + \zeta_{high} f(a_2(c_{12} - \xi)), \quad (19)$$

with

$$\zeta_{low} = [c_1 + c_2(\xi - c_3)^{c_4}] f(c_5(\xi - c_6)) + (c_7 + c_6\xi)f(c_9(c_{10} - \xi)), \quad (20)$$

$$\zeta_{high} = (a_3 + a_4\xi)f(a_5(a_6 - \xi)) + (a_7 + a_8\xi + a_9\xi^2)f(a_{10}(a_{11} - \xi)), \quad (21)$$

where a_i and c_i are constant coefficients provided in [51].

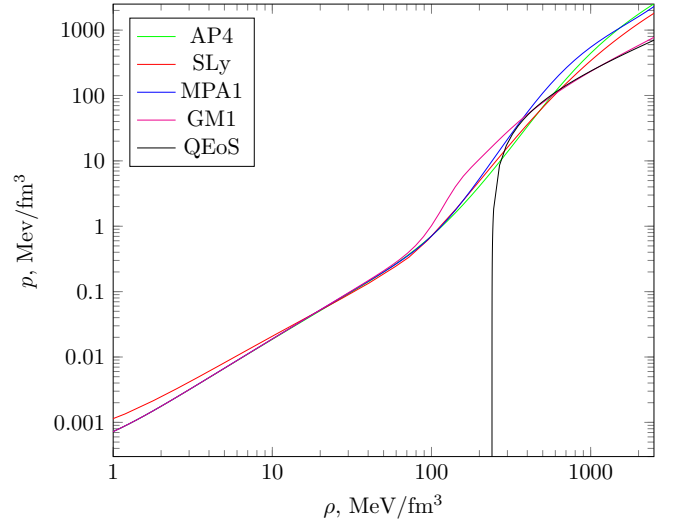


Figure 1: Equations of state for nuclear matter (AP4, SLy, MPA1, GM1) and for quark matter (QEOs) as considered in the bulk of the article.

Quark stars: Apart from the EoS described above, we have also considered the case of quark (or so called strange) stars [52, 53]. It is assumed that quark stars consist of u , d and s quarks and electrons. The deconfined quarks can form a colour superconductor. This leads to

a softer EoS with possible observable effects on the minimum allowed mass, radii, cooling behaviour and other observables. One of the simplest EoS for quark matter is provided by the so-called *MIT bag* model [54, 55] as follows:

$$p = b(\rho - 4B), \quad (22)$$

where B is the bag constant. The value of parameter b depends on both the chosen mass for the strange quark m_s and QCD coupling constant and usually varies from $b = 1/3$ ($m_s = 0$) to $b = 0.28$ ($m_s = 250$ MeV). The value of B lies in interval $0.98 < B < 1.52$ in units of $B_0 = 60$ MeV/fm³ [56]. In the following, we shall consider the case $B = 1$ and $b = 0.31$.

IV. RESULTS FOR $\alpha > 0$

Following the procedure described in Sec. II we have calculated the mass-radius and mass-central density relations for some values of parameter α for all EoS explained in Sec. III above. Results are depicted in Figs. 2, 4, 6, 8 and 10. As has been explained in the Introduction, in the context of $f(R)$ extended theories of gravity, one needs to distinguish between stellar mass bounded by star surface ($M_s = m(r_s)$) and gravitational mass as measured by a distant observer ($M = m(r \rightarrow \infty)$). Obviously, in GR these two values coincide, but in R^2 -gravity the *gravitational sphere* around the star emerges and contributes to the gravitational mass defined as $M(r_\infty)$. We have observed that for the same central density, R^2 stellar masses M_s are smaller than their GR counterparts. This decrease is then partially compensated by energetic contribution of the *gravitational sphere* when the gravitational mass M is computed. Consequently, the resulting gravitational mass-radius relation ends up to differ from the GR prediction. The *effective radius* of the *gravitational sphere* can be determined as the radius at which the metric solution is sufficiently close to the Schwarzschild solution (for example the radius at which the scalar curvature is of the order of magnitude $R \sim 10^{-5}$ in units of $r_g^{-2} \equiv c^4/G^2 M_\odot^2$). We have observed that the *effective radius* turns out to be proportional to $\alpha^{1/2}$.

On the other hand, Figs. 3, 5, 7, 9 and 11 represent the Ricci scalar curvature evolution outside the star. For R^2 gravity, the curvature turns out to be positive on star surface and decreases outside the star so that $R \rightarrow 0$ at $r \rightarrow \infty$. In the aforementioned figures, one can see the following features: Firstly the maximal gravitational mass M_{max} (defined as $dM/dr_s = 0$) of star for each EoS slowly increases with α . The value $\Delta M = M_{max}(\alpha) - M_{max}^{GR} \sim \sqrt{\alpha}$. At first glance one cannot discriminate between R^2 -gravity (for the considered values of α) and GR because such mass differences lie within typical errors of neutron star mass measurements from pulsar timing. Indeed, from Table I in [57]

one can see that 1- σ -uncertainties for neutron star masses for X-ray-optical binaries and neutron star-white dwarf binaries are $\sim 0.1 - 0.2 M_\odot$ with few exceptions. Only for neutron stars with small masses precise measurements of masses with errors $< 0.01 M_\odot$ (neutron star-neutron star binaries) are available. In conclusion, we have no well-established data on radii of these low-mass neutron stars.

Another in principle observable astrophysical quantity for these gravitational configurations is the so-called the surface gravitational redshift z_s [58], which is defined as

$$z_s = (1 - 2M_s/r_s)^{-1/2} - 1, \quad (23)$$

and therefore z_s is determined by stellar mass M_s . According to our results, in the context of R^2 -gravity for $\alpha > 0$, we conclude that $z_s < z_{s,GR}$. Moreover, the expected absolute deviation of the surface redshift from the GR value for a given mass, i.e., $\Delta z = z_{s,GR} - z_s$ turns out to be weakly dependent on the gravitational mass for typical masses of neutron stars. In principle this discrepancy could help to discriminate between GR and R^2 -gravity, and by extension to establish the validity of different classes of $f(R)$ models and other extended theories of gravity. Indeed, in the case of R^2 -gravity we have shown that $M_s < M$. Thus, the measure of the gravitational redshift would allow us to label the star by its stellar mass. On the other hand, as stated above, by means of binary systems one can measure the gravitational mass M . Accordingly, the discrepancy between these two quantities could in principle witnesses in favor of R^2 -gravity, or eventually other extended theories of gravity for which $M \neq M_s$. Unfortunately, at the present moment, one have no well-defined data about neutron-star gravitational redshifts nor radii.

Another interesting feature is the increase of star radius for a given gravitational mass close to maximal mass M_{max} when compared to GR counterparts (see Table I). In principle this fact may open the possibility to obtain an upper limit on parameter α by using observational data for neutron stars radii. Unfortunately, at the present time we have no well-established such data.

From the gravitational mass-radius relation in Figs. 2, 4, 6, 8 and 10 we see that for purely hadron EoS, the radii of neutron stars with typical masses $\sim 1.4 - 1.5 M_\odot$ hardly depends on the value of α . This effect is more noticeable for SLy and MPA1 EoS. As an illustrative example, let us mention that for GM1 EoS with inclusion of hyperons we see in Fig 8 that for several values of α the mass-radius curves intersect in the vicinity of $1.6 M_\odot$. Therefore the data corresponding to the mass-radius relation in the vicinity of either large or small masses can help to discriminate between GR and the gravitational modification under study here. Similar statements can be made for the other two neutron stars EoS.

Finally, for quark stars we have the following scenario: the density on star surface of strange stars is nonzero, namely $\rho_s = 4B$. Therefore, the scalar curvature eval-

uated at the star's surface, as predicted by GR, would be $R_s = 32\pi B$ whereas outside the star, the scalar curvature drops to zero suddenly. On the contrary, in R^2 -gravity one can construct solution with smooth decreasing of scalar curvature as can be seen in Fig. 11.

EoS	α , 10^{10} cm^2	M_{max} , M_\odot	$M_{s,max}$, M_\odot	ΔM_{max} , M_\odot	Δr_{max} , km
AP4	0	2.23	2.23	0	0
	0.5	2.22	2.18	0.045 (1.94)	0.05
	5	2.24	2.12	0.182 (1.82)	0.45
	10	2.26	2.12	0.232 (1.88)	0.63
	20	2.29	2.11	0.296 (1.91)	0.84
SLY	0	2.05	2.05	0	0
	0.5	2.05	2.02	0.042 (1.92)	0.08
	5	2.10	1.97	0.176 (1.74)	0.58
	10	2.13	1.95	0.237 (1.84)	0.80
	20	2.16	1.94	0.294 (1.86)	0.96
MPA1	0	2.49	2.49	0	0
	0.5	2.49	2.45	0.045 (2.29)	0.045
	5	2.51	2.38	0.188 (1.99)	0.55
	10	2.54	2.37	0.256 (2.01)	0.78
	20	2.57	2.34	0.320 (2.02)	1.0
GM1nph	0	1.93	1.93	0	0
	0.5	1.93	1.90	0.035 (1.88)	0.06
	5	1.96	1.80	0.169 (1.85)	0.32
	10	1.98	1.76	0.237 (1.89)	0.47
	20	2.01	1.73	0.296 (1.82)	0.60
QEoS	0	1.88	1.88	0	0
	0.5	1.89	1.82	0.072 (1.72)	0.075
	5	1.92	1.73	0.209 (1.71)	0.31
	10	1.94	1.70	0.264 (1.71)	0.41
	20	1.96	1.67	0.313 (1.82)	0.51

Table I: Parameters of compact stars for various equation of states in General Relativity (i.e., $\alpha = 0$) and for several values of α in $f(R) = R + \alpha R^2$ gravity theories. The quantity ΔM_{max} holds for the maximal contribution of the gravitational sphere outside the star in the total value of the gravitational mass (having included the stellar mass). On the fourth column, the value for the corresponding gravitational mass is given in brackets. In the last column we show the maximum difference in the star radius as compared with the General Relativity counterpart. This maximum difference takes place for a star with a mass equal to the maximal mass in General Relativity for a given equation of state.

V. RESULTS FOR $\alpha < 0$

In this Section we have considered the case $\alpha = -0.05 \text{ km}^2$ as an illustrative example in order to get a better understanding of the behaviour of the system of Eqs. (5)-(8) when applied to R^2 -gravity models with negative parameter α . The considered central densities for the studied examples below correspond to stars possessing a GR maximal mass for each of the chosen EoS. The system (5)-(8) is thus integrated up to a large enough distance ($r \approx 3 \cdot 10^5 \text{ km}$).

We have analysed the $B(r)$ dependence with the radial

coordinate and studied the behaviour of this metric function as $r \rightarrow \infty$ so by the use the Eq. (17) we have tried to extract the corresponding gravitational mass. Another (but completely equivalent) way we have followed consisted of trying to find such an initial value $B(0)$ for which the function $B(r) \rightarrow 1$ as $r \rightarrow \infty$. We shall designate this value as $\bar{B}(0)$. For the latter method, one can consider a sequence of $\bar{B}_i(0)$ values such that $B(r_i) = 1$. Then extrapolating this sequence for $r_i \rightarrow \infty$ gives the required $\bar{B}(0)$. The gravitational mass can then be calculated according to Eq. (15).

We have also performed the following calculations: after having assumed that $m(r)$ varies slowly at large distances within some interval Δr (we take $\Delta r \approx 10^4 \text{ km}$), we have approximated $B(r)$ by a functional dependence

$$B(r) = B_\infty \left(1 - \frac{2m}{r}\right) \quad (24)$$

in this Δr interval. Thus, as a rough estimation one can identify the corresponding mass for given r with m . For GR this scheme gives correct value for gravitational mass, whereas for R^2 gravity our method shows that the found mass grows with the radial distance.

In order to illustrate the reasoning, we have chosen $B(0) = 0.1$. Table II below summarises the found results for the MPA1 EoS case. We have also calculated the values $\bar{B}_i(0)$ at which for r_i the condition $1 - B(r_i) = \beta$, is accomplished with $\beta = 10^{-4}$. In GR $B(r)$ exactly scales like $1/r$ and therefore one can extract the gravitational mass M using dependence $B(r)$ for an arbitrary $B(0)$ value, by simply approximating this dependence to the well-known function $B_\infty(1 - 2M/r)$. For other theories beyond GR, provided $m(r)$ shows an almost-constant plateau at large distances, an asymptotic mass can be extracted. In particular, for $f(R) = R + \alpha R^2$ ($\alpha < 0$) one can assume that the aforementioned plateau on the function $m(r)$ evolution will be found at sufficiently large distances. Nonetheless, our method shows that the approximation of $\bar{B}(0)$ (or equivalently B_∞ for an arbitrary $B(0)$) strongly depends on the considered interval of distances.

Specifically we have considered the approximation of $B(r)$ by a function of the form $B_\infty(1 - 2m/r)$ in intervals $r_i < r < r_i + 10^4 \text{ km}$ for some r_i and results have been shown in Table III. In this case the dependence $(1 - B(r)/B_\infty)r/2$ has a maximum near r_i . We have also considered the interval $10^3 \text{ km} < r < 5 \cdot 10^4 \text{ km}$ for $B(r)$. In this case we have for $B_\infty = 1.488271$ and the maximal value for $m(r)$ is $2.71M_\odot$. If for instance the consider distance interval is now $2 \cdot 10^4 \text{ km} < r < 7 \cdot 10^4 \text{ km}$ one obtains $B_\infty = 1.488362$ and maximal of $m(r)$ is $3.15M_\odot$. Thus our method leads us to conclude that $B_\infty - B(r) \approx r^{-1}$ even for large distances. Consequently, the main point of our analysis is that the function $B_\infty - B(r)$ (or $1 - B(r)$ for calibrated initial condition for $B(0)$) tends to 0 more slowly than r^{-1} at $r \rightarrow \infty$. Therefore we are led to conclude that we have

$r_i, 10^4 \text{ km}$	B_i	$\bar{B}_i(0)$	$m(r_i)$	$B_{i, \text{GR}}$
1	1.487088	0.067178	2.64	1.482622
2	1.487687	0.067151	2.83	1.483165
3	1.487900	0.067141	3.02	1.483346
4	1.488013	0.067136	3.19	1.483437
5	1.488085	0.067133	3.37	1.483491
6	1.488135	0.067131	3.55	1.483528
7	1.488173	0.067129	3.72	1.483553
8	1.488203	0.067128	3.90	1.483573
9	1.488227	0.067127	4.07	1.483588
10	1.488248	0.067126	4.25	1.483600
12	1.488280	0.067124	4.60	1.483618
14	1.488305	0.067123	4.95	1.483631

Table II: Calculation of $B(r_i) \equiv B_i$ at some r_i for $B(0) = 0.1$ and MPA1 equation of state. The fourth column shows $m(r_i)$. Radial distances have been considered in the interval $r_i - 5 \cdot 10^3 \text{ km} < r < r_i + 5 \cdot 10^3 \text{ km}$. As observed, masses $m(r_i)$ grow with r_i . As expected, numerical calculations in General Relativity give the well-known result, i.e., the function $(B_\infty - B(r))r/2$ remains constant being the gravitational mass is $2.48M_\odot$ for this case.

no well-defined limit of mass since the function $m(r)$ seems to grow with radial distance. In fact the value of $\bar{B}(0)$ (or B_∞) decreases (increases) with growing of maximal limit of radial interval under consideration.

Of course one can assume that the results above only take place for some EoS only. In order to see if this is true, we have considered all the other three-nucleon EoS from Sec. III, namely AP4, SLy and GM1. For those, we have obtained that the $m(r)$ growth rate with the radial coordinate slightly depends on the specific EoS, but there is no qualitative difference with respect to the trend fully explained above.

$r_i, 10^4 \text{ km}$	B_∞	$\bar{B}(0)$	m_{max}
1	1.488284	0.067192	2.74
2	1.488325	0.067190	2.92
3	1.488352	0.067189	3.05
4	1.488372	0.067188	3.27
5	1.488387	0.067187	3.44
6	1.488400	0.067186	3.62
$0.1 < r < 5$	1.488271	0.067919	2.71
$2 < r < 7$	1.488362	0.067188	3.18

Table III: Results for B_∞ in the interval $r_i < r < r_i + 10^4 \text{ km}$ for $B(0) = 0.1$ for the MPA1 equation of state and corresponding of $\bar{B}(0)$ in same interval. We give the maximum value of $m(r)$ (m_{max}) for corresponding the \bar{B} and B_∞ .

VI. CONCLUSIONS

In this paper we have investigated the main features and existence of realistic models of compact neutron and quarks stars in one paradigmatic extension of General Relativity. In other words we have studied these static and spherically symmetric configurations

for a class of fourth-order $f(R)$ extended theories of gravity, whose gravitational Lagrangian adopts the form $f(R) = R + \alpha R^2$. As explained in the bulk of the article, the resolution of the generalised system of Tolman-Oppenheimer-Volkoff equations for positive α has indeed provided the existence of solutions, whereas for negative α it has been impossible to recover a weak-field (Newtonian) limit far away from the star, concluding the impossibility of stable configurations in such a region of the parameter space.

Note that our analysis has not considered any perturbative approximation whatsoever when dealing with the system of equations resolution nor a shooting method has been required owing to the chosen variables. Thus, we have found that for the same central density the stellar mass as bounded by the star surface decreases when the parameter α increases. This result can be understood as a proof that in extended theories of gravity, a more intense gravitational interaction leads to stable stars which for the same mass in Einsteinian gravity would either not exist or be condemned to be black holes. Also, we have seen that, unlike previous works [59] which naively defined the mass as the matter integral over the star volume, we have shown how a more careful matching of interior and exterior regions causes the total gravitational mass to be unbounded for sufficiently large values of α (with specific values depending on the equation of state under consideration). Consequently this more general consideration of conditions to be imposed at the edge of the star is twofold.

First, these reasonable matching conditions imply that the scalar curvature is not identically null on the edge of the star, even so (for $\alpha > 0$) it is exponentially suppressed and goes to zero and accordingly asymptotic flatness is eventually recovered. This can be thought of as being equivalent to the existence of an additional energetic content around the star which might be assigned to the additional extra scalar mode present in $f(R)$ theories. It is precisely this mode which can help to prevent the gravitational collapse and increase the gravitational mass as observed by a distant observer. Thus in the event of merging objects of this kind, the available energy would be higher than General Relativity counterparts and the emission of gravitational waves from such a configuration might be feasible and detectable by present and future interferometers [31, 60–64].

Secondly, we have indeed corroborated the existence of exterior static and spherically symmetric physical solutions differing from the usual Schwarzschild solution (non Ricci flat) provided $f(R)$ theories are assumed to represent the underlying gravitational theory in high-curvature environments. This fact constitutes a straightforward violation of the Jebsen-Birkhoff theorem in $f(R)$ theories [42, 65]. Hence it seems that the most

natural spacetime solutions as generated by a spherical stable neutron or quark star is not the Schwarzschild solution once Einsteinian gravity is replaced by simple extensions of the standard Einstein-Hilbert Lagrangian.

Concerning the observational imprints to be expected from our results, let us claim that the net effect on the total mass is a faint, although α -parameter, dependent increase in the mass as perceived by a distant observer with respect to the General Relativity prediction for the same central density and equation of state. Also, we have concluded the increase of star radii in the vicinity of maximal masses as well as the radii reduction for stars with masses of the order of the Sun mass. Then, we can think of at least two ways for discriminating between Einsteinian gravity and possible modifications from scalar-tensor theories, and by extension other extended theories of gravity with analogous effects. Firstly, high-accuracy measurements of surface redshift can shed some light on the underlying theory. Indeed, for the same asymptotic gravitational mass, the pure stellar mass bounded by surface as generated by R^2 -gravity would be smaller than the Einsteinian value and therefore the measured surface redshift would be smaller than predicted. Secondly, another observable effect would be the determination of gravitational maximal masses, which we have calculated for competitive both neutron plasma and quark stars, using X-ray-optical binaries and neutron star-white dwarf binaries. Notwithstanding, the present precision in mass measurements is not enough to discriminate between competing theories yet. Both methods have been shown here as competitive proposals to break the degeneracy problem afflicting alternative theories of gravity.

Finally, for $\alpha < 0$ our analysis has shown that outside the star the additional energetic effect explained above, i.e., the appearance of a *gravitational sphere*,

is also present. Nevertheless, the scalar curvature suffers damped oscillations instead of an exponential attenuation. Moreover, in these cases the contribution of the *gravitational sphere* into the gravitational mass has been shown to increase indefinitely with radial distance. Therefore we conclude that R^2 -gravity with $\alpha < 0$ predictions are not consistent with the existence of realistic models of compact relativistic stars, at least for the studied equations of state. The inadmissibility of R^2 gravity with $\alpha < 0$ was made earlier in cosmological contexts so our result shows that these classes of models would not be acceptable in the astrophysical context either.

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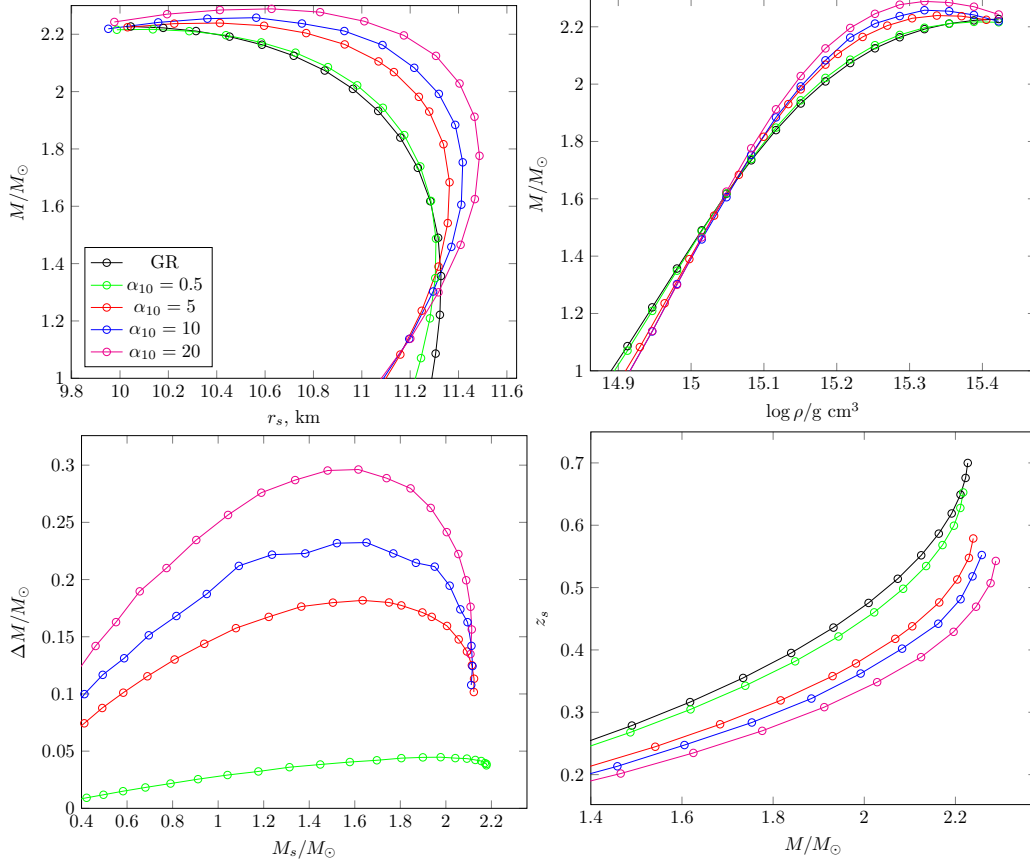


Figure 2: Upper panel: relation between the gravitational mass (M) and radius of the star for AP4 EoS in R^2 -gravity when compared with the General Relativity predictions (left); relation between the gravitational mass and the central density of the star (right). The symbol α_{10} refers to $\alpha \times 10^{10} \text{ cm}^2$. Lower panel: the dependence of the contribution of gravitational sphere into gravitational mass $\Delta M = M - M_s$ with stellar mass $M_s \equiv m(r_s)$ (left); surface redshift as a function of gravitational mass (right).

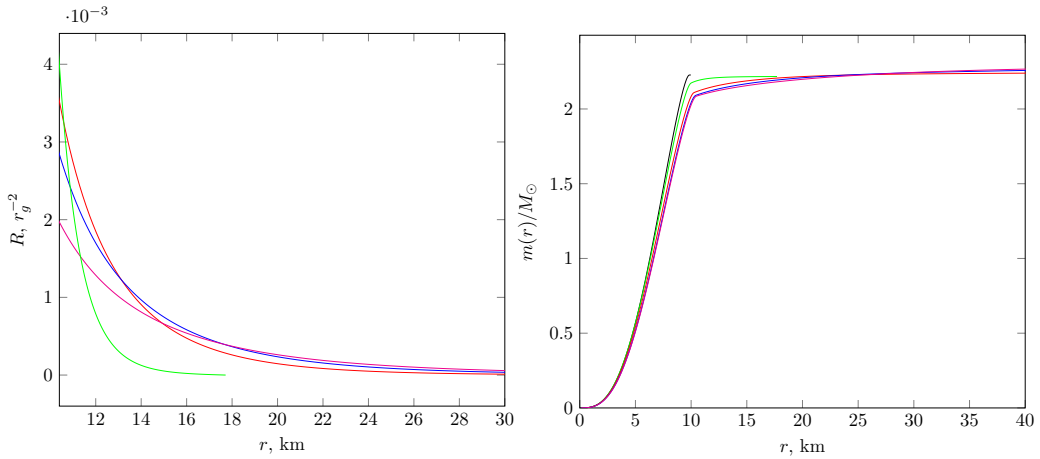


Figure 3: Left panel: Dependence of the scalar curvature outside the star for stellar configurations with maximal mass for AP4 EoS. Scalar curvature is given in units of $r_g^{-2} \equiv c^4/G^2 M_\odot^2$. Right panel: the stellar mass profile $m(r)$ for stellar configurations with maximal gravitational mass.

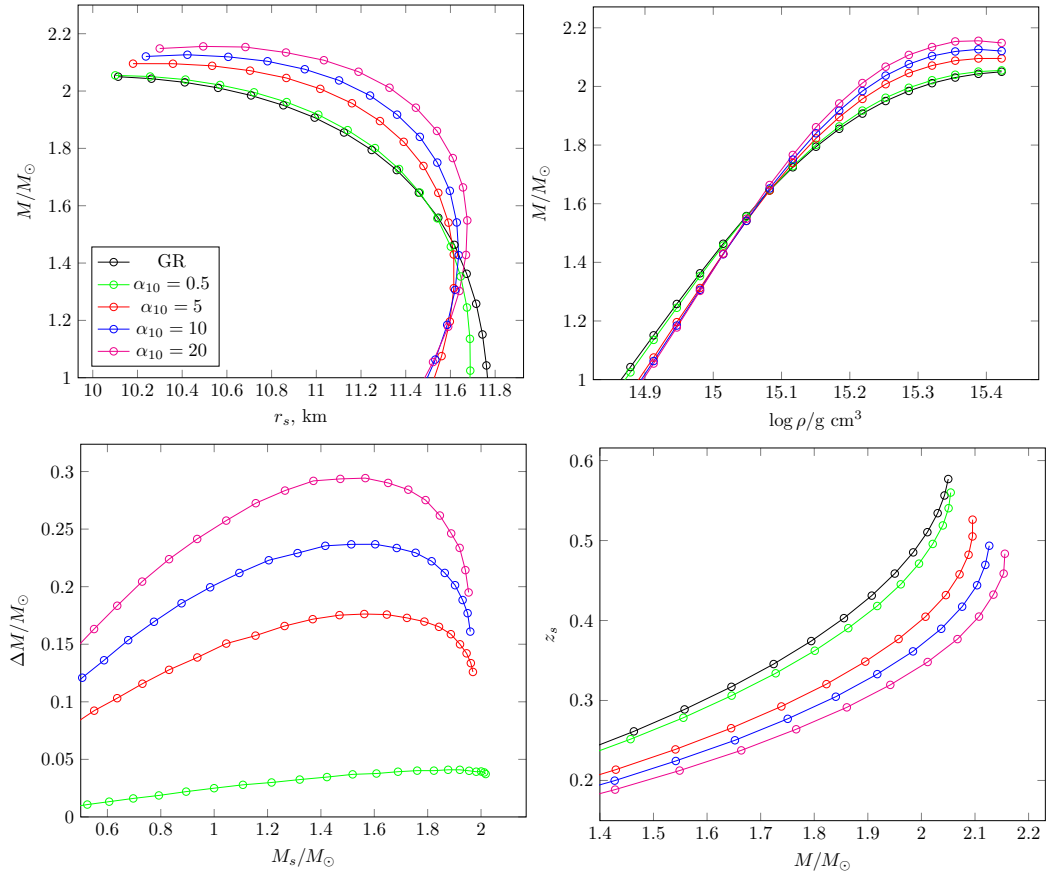


Figure 4: The same as on Fig. 2 but for SLy EoS.

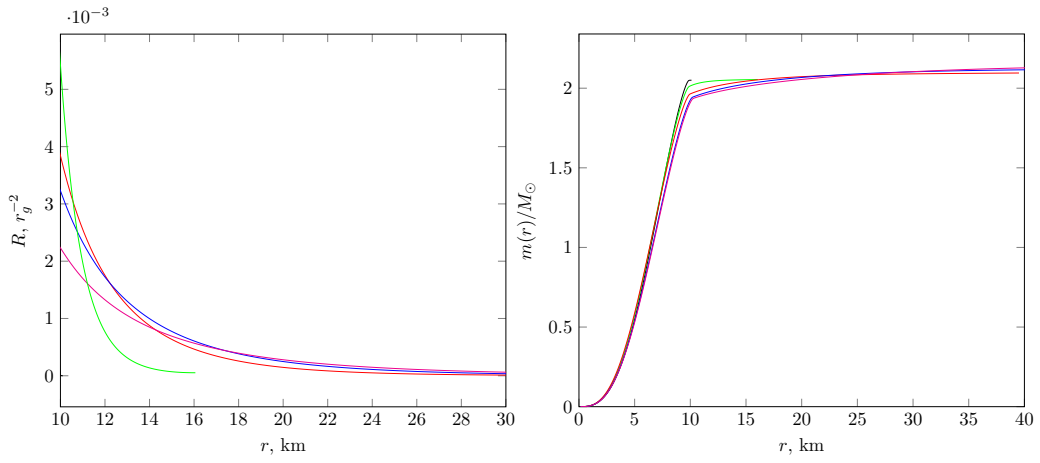


Figure 5: The same as on Fig. 3 but for SLy EoS.

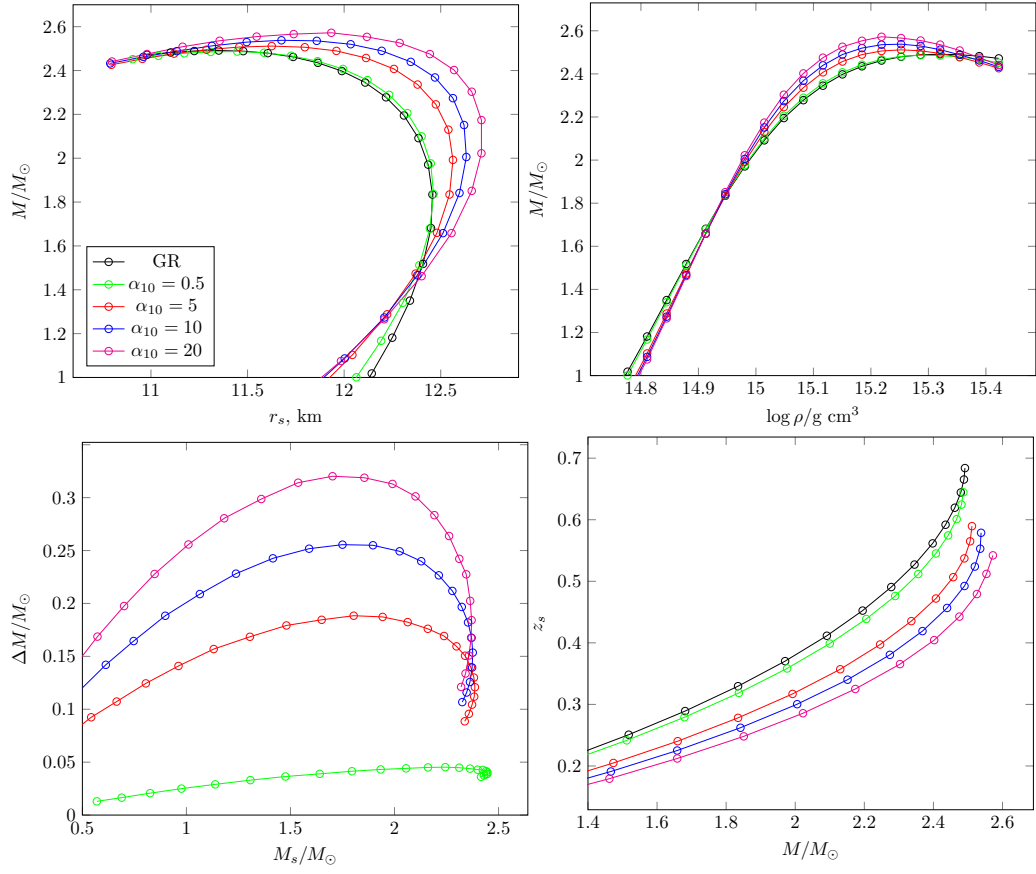


Figure 6: The same as on Fig. 2 but for stiff MPA1 EoS.

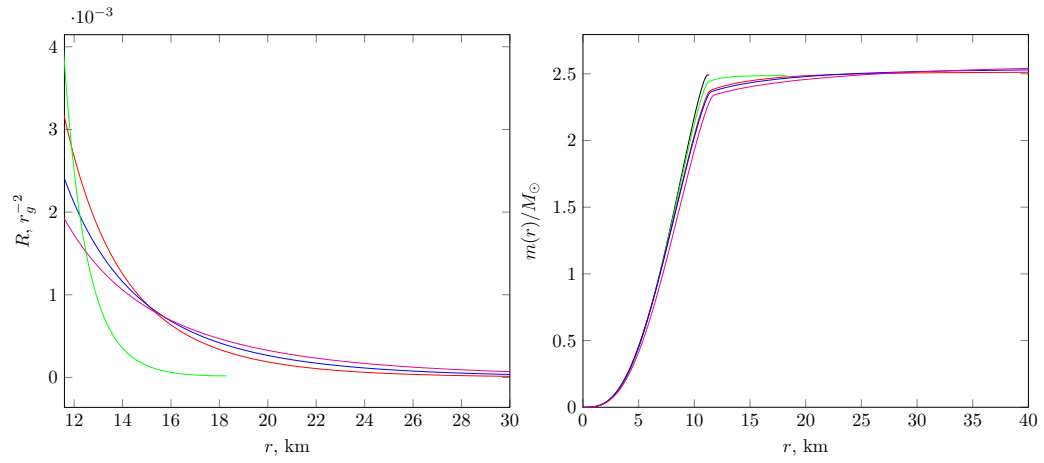


Figure 7: The same as on Fig. 3 but for stiff MPA1 EoS.

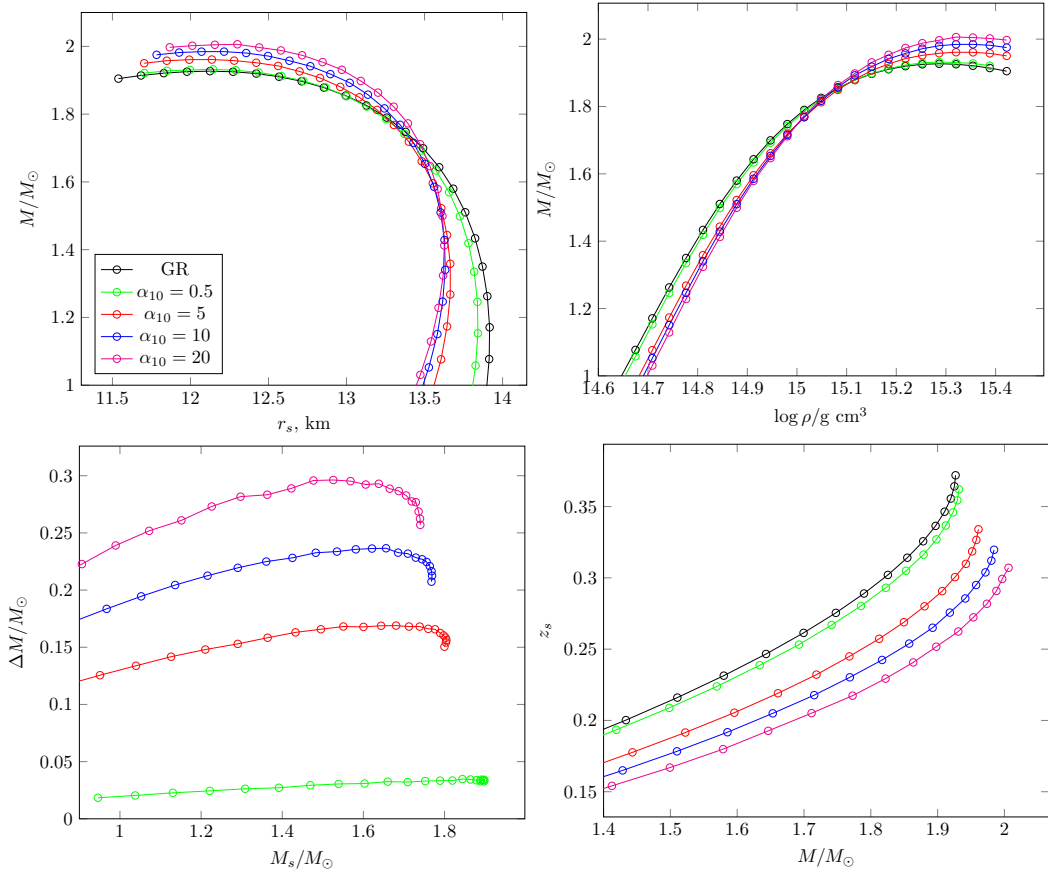


Figure 8: The same as on Fig. 2 but for GM1 EoS with inclusion of hyperons.

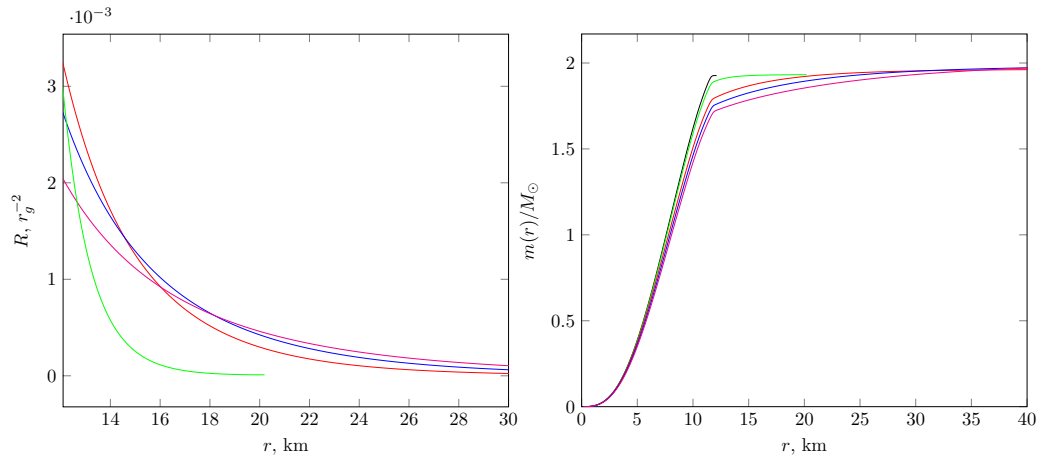


Figure 9: The same as on Fig. 3 but for GM1 EoS with inclusion of hyperons.

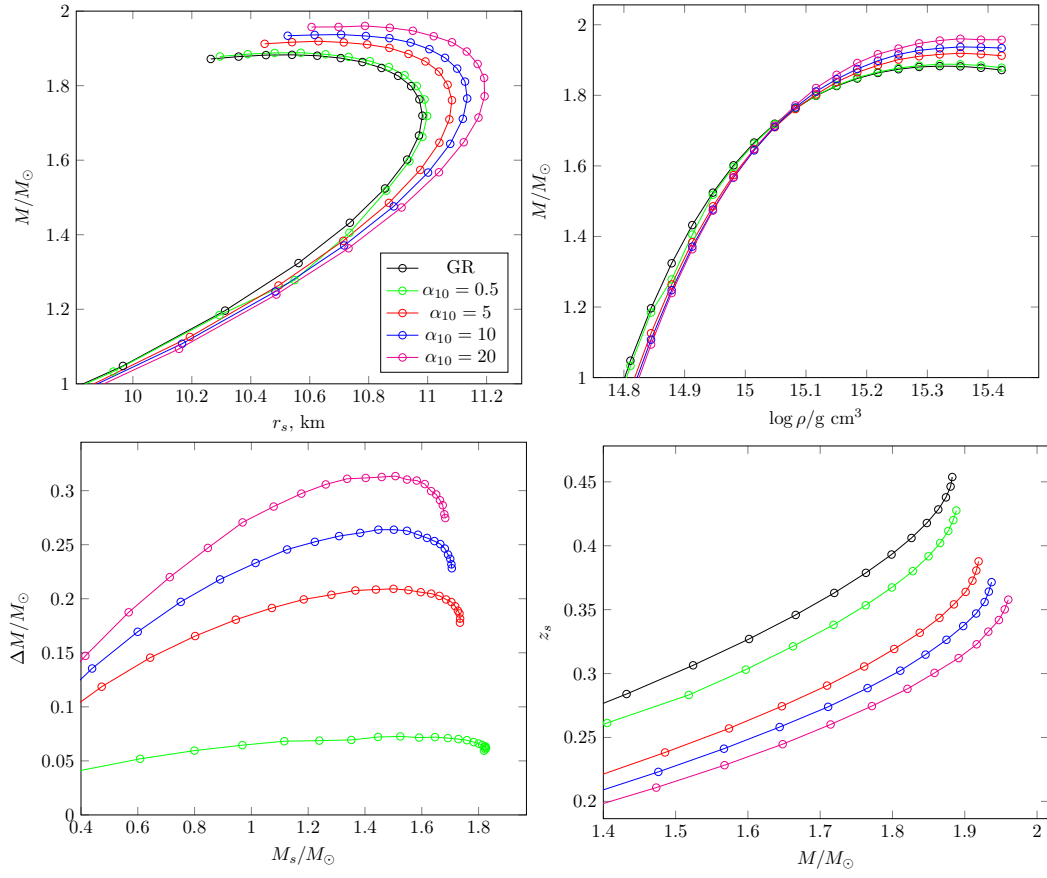


Figure 10: Upper panel: relation between gravitational mass M and radius for simple quark EoS for R^2 -gravity in comparison with General Relativity (left); relation between gravitational mass and central density of the star. Lower panel: the dependence of the contribution of gravitational sphere into gravitational mass $\Delta M = M - M_s$ from M_s (left); surface redshift as a function of gravitational mass (right).

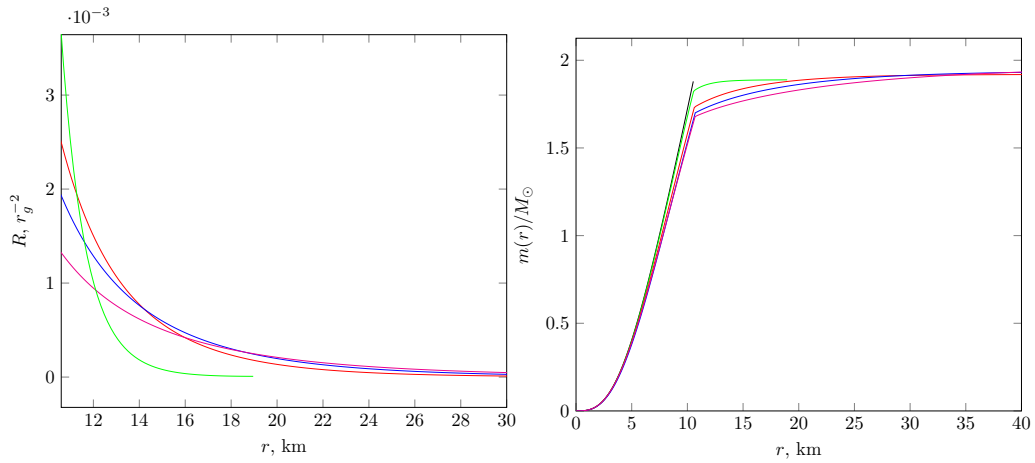


Figure 11: Left panel: The dependence of scalar curvature outside of star for stellar configuration with maximal mass for simple quark EoS. Right panel: the stellar mass profile $m(r)$ for stellar configurations with maximal gravitational mass.